Schopenhauer Diagrams for Conceptual Analysis

Michał Dobrzański[1] [0000-0002-0718-643X] and Jens Lemanski[2][0000-0003-3661-4752]

[1] Institute of Philosophy, University of Warsaw, Poland
[2] Institute of Philosophy, University of Hagen, Germany
michaldobrzanski@uw.edu.pl

Abstract. In his *Berlin Lectures* of the 1820s, the German philosopher Arthur Schopenhauer (1788–1860) used spatial logic diagrams for philosophy of language. These logic diagrams were applied to many areas of semantics and pragmatics, such as theories of concept formation, concept development, translation theory, clarification of conceptual disputes, etc. In this paper we first introduce the basic principles of Schopenhauer's philosophy of language and his diagrammatic method. Since Schopenhauer often gives little information about how the individual diagrams are to be understood, we then make the attempt to reconstruct, specify and further develop one diagram type for the field of conceptual analysis.

Keywords: spatial logic diagrams, linguistic abstraction, bundle theory.

1 Introduction

It is only in recent years that it has become known that in his so-called *Berlin Lectures* of the 1820s [1] the German philosopher Arthur Schopenhauer made intensive use of logic diagrams in an original and novel way. One of Schopenhauer's original ideas was to use diagrams not only for logic and eristics, but also for semantics and conceptual analysis.

We would like to present and elaborate on this idea in the present paper. For this purpose, we will first introduce the basic principles of Schopenhauer's theory of language in Section 2, illustrating them with some of his logic diagrams for conceptual analysis. Since Schopenhauer himself gives only a few sentences about the use of these diagrams, we will reconstruct and develop one type of diagram in Section 3 and explain the basic principles presented in Section 2 with the help of this reconstruction.

To avoid misunderstandings, it should be noted that we do not claim that Schopenhauer diagrams are better than other visual systems, such as conceptual graphs [2], concept diagrams [3] etc., which are currently used in semantics. Although the *Berlin Lectures'* diagrams show similarities with Euler, Kant, Venn and Peirce diagrams, we will avoid comparisons with other diagram systems as far as possible and therefore use the neutral term "Schopenhauer diagrams" here. In addition, we will leave out many topics, theses and problems of Schopenhauer's theory of language[1], but—if possible—we will refer to existing literature in several places.

2 Schopenhauer's Theory of Language

In this section, we give an overview of Schopenhauer's theory of language taken from the *Lectures on the Entire Philosophy*, Chapter 3, which is entitled *Of the Abstract Representation, or Thinking: which Chapter contains Logic*. In §§1–6 we will focus on Schopenhauer's theory of representation and concepts, in §§7–10 on his logic and in §§11–14 on some visualizations of concepts.

§1. Definition of Concepts. Schopenhauer distinguishes two classes of objects that can be perceived by the subject, which he calls representations: (1) intuitive representations that are recognized by the external senses and (2) abstract representations recognized by reason alone ([1], 118) that are free of temporal-spatial determinations. For Schopenhauer, the representations of class (2) are concepts. As products of reason, they have a close connection with language, which is described as one of the "main expressions of reason" in man ([1], 240f.). Concepts are the actual material of human thinking, or, in other words, thinking and reasoning is described only as the "realization of concepts" (Vergegenwärtigung der Begriffe; [1], 243).

§2. Concreta and Abstracta. Schopenhauer divides concepts into *concreta* and *abstracta*. *Concreta* are "abstracted directly from intuitive representations". In contrast, *abstracta* are formed by omitting some properties of other concepts. Examples of *concreta* are concepts such as `blue`, `dog`, `house`, whereas *abstracta* is used for concepts such as `quality`, `artwork`, `friendship` ([1]; 252). Despite this distinction, Schopenhauer points out that, strictly speaking, all concepts are abstract and the distinction between *abstracta* and *concreta* is only useful to clarify the relation of concepts to each other, but not the relation of concepts to intuition.

§3. Generality of Concepts. Similar to Euler ([7], L. CI), Schopenhauer also denies the possibility of singular propositions (*propositio singularis*). He claims that "a concept is always general, even if there is only one thing that is thought by it; and only a singular intuition that gives it content (Gehalt), is a proof of it", since "the concept is always an abstractum, a thought, but never a single individual thing". This is true even for proper names such as "Socrates", since it is also possible to denote more than one object with it ([1], 276f.).

§4. Origin of Concepts. According to Schopenhauer, reason produces concepts by abstracting from the many properties of objects that are given in intuitive representation. Thus, a concept "does not contain everything" that is given or contained in its intuitive basis. On the other hand, "innumerable intuitive objects" can be thought of with the help of a concept ([1], 249). However, Schopenhauer emphasizes the dependence of concepts on intuition: "the whole world of reflection [...] rests on the intuitive one as its basis of cognition" ([1], 252).

[1] For these topics, see [4], [5], [6].

§5. Empirical Criterion of Meaning. He also claims that each concept can be described as distinct and meaningful if and only if, in the course of concept analysis, its properties can ultimately be substantiated with clear intuition ([1], 254f.). Thus, *abstracta* must be broken down to *concreta*, and *concreta* must refer to given objects in intuitive representations. This goes along with his rejection of *a priori* concepts and his criticism of innatism ([1], 235). This empirical criterion of meaning thereby also forms the basis for his criticism of scholasticism ([1], 255), rationalism ([1], 254) and idealism ([1], 236f., 495).

§6. Incommensurability of Language and Thought. Furthermore, he seems to claim the separation of conceptual thought processes from language, despite the close connection of concepts to words. Words are described as merely sensory "sign[s] of concept[s]" ([1], 243). They are, however, necessary in order to remember concepts willingly (willkürlich) and to be able to perform intersubjectively perceptible thought operations with them. Thus it is not possible to communicate a concept for which there is no word ([1], 244). At the same time there is no isomorphism between language and thinking. Schopenhauer makes it clear in numerous written passages that one should not equate language analysis with concept analysis: It would be wrong "if the argument that signs are necessary for concepts was used to justify the assumption that we would actually operate with the signs alone when thinking and talking, and that they completely represent the concepts" ([1], 247). This is still based on Schopenhauer's strict separation of two types of representation—the intuitive (temporal-spatial) and the abstract. Words belong to the first, since they are just sensory perceptible signs of abstract thoughts. Schopenhauer, by the way, sees his "sharp separation of concepts from intuitive representation, i.e. things" as an important contribution to the history of logic ([1], 357).

§7. Definition of Logic. According to Schopenhauer, logic is "the general knowledge of the peculiar way of proceeding of reason, gained through the self-observation of reason and abstraction of all content, expressed in the form of rules" ([1], 362). It is further described as the discipline that deals with the analysis (of the operations) of concepts, i.e. thinking and reasoning, or "the pure science of reason", which mainly teaches how one may "operate" with concepts ([1], 368). Logic need not necessarily have anything to do with language; however, both language and logic have in common that they must use signs to represent thoughts. Since language and logic have different rules and since language is only understood as a system of sensory perceivable signs for the evocation of concepts, it is possible that other signs could also be used for both purposes, for example: diagrams.

§8. Extension and Intension. Schopenhauer introduces his circle diagrams by claiming that concepts have a "sphere" (Sphäre) or a "circumference" (Umfang). Because of the sphere and the circumference, concepts are limited and bounded. Thus, expressions such as "boundary", "circumference" and "sphere" refer to a limited set of objects (intuitive or abstract) that are thought of in a concept ([1], 257)—nowadays we would usually call this the extension of a concept. Furthermore, Schopenhauer also speaks about the content (Inhalt) of concepts in order to denote the given properties (Merkmale) of a represented object—this could be understood as the intension of a concept.

In Schopenhauer's words: The circumference is equal to what can be thought "through" a concept, and the content with what can be thought "in" a concept ([1], 258).

§9. Law of Reciprocity. The relationship between extension and intention is stated in the law of reciprocity. According to Schopenhauer, the circumference of the sphere of a concept is in an "inverse relationship" to its content ([1], 258). In other words: The more extension a concept has, the less intension it has and vice versa. This law, which was made popular by Kantian logic, and especially the problems with the notion of intension, are discussed in great detail by Hauswald [8].

§10. Bundle-Theory. What Schopenhauer seems to mean by content (intension) is a bundle of concrete properties associated with a concept. One can deduce this from the law of reciprocity: the wider the sphere of a concept is or the more universally a concept is applicable to different objects, the smaller the bundle of concrete properties that describe the various objects. This is supported by the statement that the concept that has the most content is the one in which we think the "most properties" ([1], 271). Content, as a bundle of properties, is thus one of the features that can be illustrated by specific types of diagrams.

§11. Conceptual Spheres. For Schopenhauer, conceptual spheres are the actual substance of logic—a discipline of reason that deals with the correct "cognition of the relationships of conceptual spheres to one another" ([1], 364). With that Schopenhauer anticipates what many current authors also explain: that these relations, and indeed all possible ones, can be represented by diagrams in the form of circles and that there is a kind of isomorphic relation between circle diagrams and conceptual spheres (i.e. human thoughts!). Where this isomorphism comes from cannot be explained by Schopenhauer; however, he acknowledges that this is "an extremely fortunate occurrence" and states that it was made popular by Gottfried Ploucquet (in square form), Johann Heinrich Lambert (in line form) and Leonhard Euler (in circle form) ([1], 269). Circles symbolize conceptual spheres and not words, since Schopenhauer always consistently speaks of "conceptual spheres" (see [1], 269 ff.). It should be noted that words must nevertheless be used in the diagrams to designate the concepts that are actually to be examined with them. It seems, then, that Schopenhauer's diagrams should primarily be understood as the study of human thought that takes place in concepts and is a process largely independent of language. However, language is needed to demonstrate it.

§12. Circle-Inclusion. But how should the diagrams be read? Schopenhauer is curt. On one hand he explicitly states that the "relative size of the spheres", i.e. the size of one sphere in relation to another, refers "not to the size of the content of the concept, but to the size of the circumference" ([1], 271). On the other hand, a number of diagrams can be found in the *Lectures* where the size of the circle seems to be irrelevant. Even though this needs further analysis, it can at least be assumed that it is necessary that two circles have different sizes if one conceptual sphere is completely contained in another. For example: The concept `triangle` (Dreieck) has more concrete properties than the concept `figure` (Figur) but a figure comprises more objects than just triangles, thus the narrower conceptual sphere of `triangle` is completely included in the wider sphere of `figure`. The same applies to the concept `bird` (Vogel) in the concept `animal` (Thier), as Fig. 1 illustrates.

Fig. 1 ([1], 258)

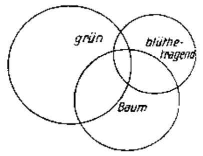


Fig. 2 ([1], 257)

§13. Circle-Exclusion: However, in other situations Schopenhauer seems to treat the above stated rule more loosely, depending on what is he intends to demonstrate with the diagram. For example: the size of two circles does not play a role if two concepts have nothing in common and are mutually exclusive. The spheres of concepts such as `stone` and `animal` have no common extension. This means, that there is no object in intuitive representation which can be both, a stone and an animal. Therefore, two circles with arbitrary sizes have to be drawn in the diagram, and both show neither an intersection nor an inclusion ([1], 274). The same applies e.g. to `triangle` and `bird` in Fig 1.

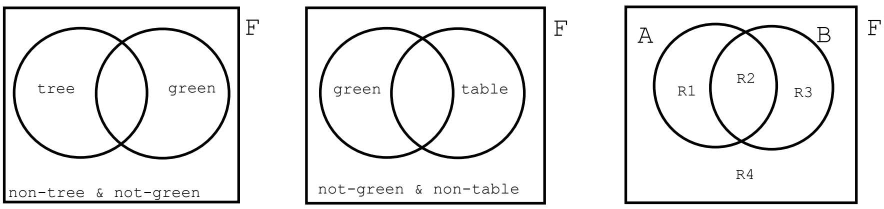
§14. Circle-Intersection: Schopenhauer also depicts concepts that "mutually contain each other". In this case, the content (not the circumference) of one results directly from the content of the other (see [1], 273). In summary, it can be assumed that in the diagrams the relationship of the conceptual contents (intension) is represented by the spatial relationship of the circles to each other, while the size of the circles only sometimes might represent the conceptual circumference (extension). In Fig. 2, for example, we see three concepts denoted by the words `green` (grün), `tree` (Baum) and `flower-bearing` (blüthetragend), whose mutual containment is represented only by the spatial relations of the circles.

3 Schopenhauer Diagrams

Fig. 2 is a good example of how Schopenhauer finds new applications for logic diagrams in the *Berlin Lectures*. In this case, the circle diagram is applied to conceptual analysis but only explained with a few sentences in the text. However, we are not concerned here with interpretation of these few sentences, but with the reconstruction, specification and development of Schopenhauer's diagrammatic ideas in semantics. As a case study, we take Fig. 2 as a paradigm and reconstruct in §§15-18 a diagram with two concepts from the intuitive representation. In §§19-25, we end with a reconstruction of diagrams with three and more concepts.

§15. First Two Definite Concepts. Let us imagine that in our intuitive representation we find a certain object that we occupy with the concept `tree`. Since it has been abstracted directly from the intuitive representation (§2), the concept is a *concretum* which has a definite sphere (§§8, 11) and is illustrated by a circle. Furthermore, we have found criteria through our intuitive representation that allow us to say what belongs to the tree and what does not. Thus, we can also refer to the indefinite concept `non-tree` which is located outside the circle of `tree` but inside a square frame `F`. Both diagrammatic objects, the circle and the frame together form diagram D1. But let us now assume that we find in the intuitive representation another object called `table` that intuitively has similarities to and differences from the first object mentioned. For `table` and `non-table` we can therefore draw a similar diagram D1*. Let us further assume that both objects of intuitive representation have a common property which we call `green`. In this case, we can now make an addition for D1 and D1* and draw a further conceptual sphere which is marked with the word `green`.

§16. Diagrammatic Representation. But where exactly is the conceptual sphere of `green` in D1 or D1*? Since both objects intuitively have at least one thing in common—`green`—but also have differences, `green` cannot be congruent with `tree` in D1 or with `table` in D1*. Circle-Inclusion (§12) is therefore not possible. However, since `green` is assigned to both concepts, otherwise the similarity of the property would not be intuitively given, `green` must have an intersection with `tree` or `table`. Thus, Circle-Exclusion (§13) is also excluded. This means that a part of `green` is contained in the region of `tree` and a part of `green` in the region of `non-tree`, and this region of `non-tree` will also contain `table` somewhere. So the relationship of `green` to `tree` and `table` is that they mutually contain each other. The result of this consideration is that green divides the sphere given in D1 and D1* respectively, i.e. D2 and D2*. We see in D2 and D2* that the indefinite concept is located outside both circles and thus negates the two definite concepts. However, the intuitive representation given in D1 and D1* now appears to be separated: the circle, which in D1 and D1* denoted one object with many properties, has now been divided into two regions. This raises the question of which region of a diagram such as D2 or D2* is closest to intuitive representation.

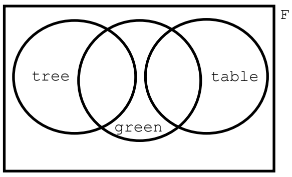


D2 D2* D3

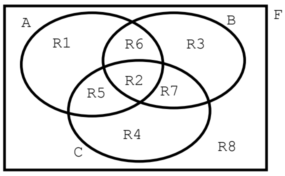
§17. Syntax of D3. In diagram D3, we separate the syntax of the diagrams from the semantics and therefore assign the concepts in D2 and D2* the variables `A` and `B`. Within the diagrammatic frame (`F`), D3 shows two circles (`A`, `B`), which together form four areas that can be called regions (`R1`, `R2`, `R3`, `R4`). Connections of regions can form diagrammatic objects, such as `{R1, R2} = A` or `{R2, R3} = B`. Similar to [9], the distinction between diagrammatic objects and regions results in several options for describing D3 such as: `{R1}` depicts the abstraction of `B` from `A`, $A \setminus B$. `{R2}` depicts the intersection of `A` and `B`: $A \cap B$. `{R3}` depicts the abstraction of `A` from `B`: $B \setminus A$. `{R1, R2, R3}` depicts the union of `A` and `B`: $A \cup B$.

§18. Semantics of D1 or D2. D2 and D2* display two circles, four regions and a diagrammatic frame in the same setting as D3. Thus, we can substitute `tree`, green or table with A or B. For example, we take D2: {R1} depicts the abstraction of green from tree: tree \ green. {R2} depicts the intersection of tree and green: tree ∩ green. {R3} depicts the abstraction of tree from green: green \ tree. {R1, R2, R3} depicts the union of tree and green: tree ∪ green. {R4} depicts the negation of tree and green: F \ (tree ∪ green).

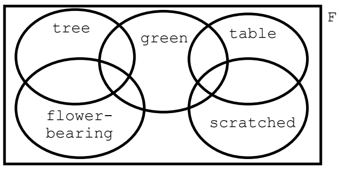
§19. n-term Diagrams. Schopenhauer himself has also designed diagrams for n-terms, which produce large conceptual clusters by circle intersection and exclusion. As an example, one could take the concepts tree and table, which according to §15 are different objects (tree △ table), but both can have the property green. A unification of D2 and D2* would then be D4.



D4 (D2+D2*)



D5



D6

§20. Regions of D5. But if there is no exclusion for three concepts, we arrive at D5: {R1}: $A \setminus (B \cup C)$; {R2}: $(A \cap B \cap C)$; {R3}: $B \setminus (A \cup C)$; {R4}: $C \setminus (A \cup B)$; {R5}: $(A \cap C) \setminus B$; {R6}: $(A \cap B) \setminus C$; {R7}: $(B \cap C) \setminus A$; {R8}: $F \setminus (A \cup B \cup C)$.

§21. Semantics for D5. We now adopt the semantics of D2, so that A denotes the concept tree and B the concept green. Let us now assume that the object of intuitive representation of §15 also has the property of bearing a flower. We now use the concept flower-bearing for C and thus arrive at a semantics for the respective regions in which, for example, {R5} denotes an object to which the concepts green, flower-bearing apply, {R6} on the other hand, designates objects with the properties tree and green. Thus, D5 gives the syntax for Fig. 2.

§22. Bundle of Intersections According to the arguments of §§2–4, *concreta* are concepts that have no or as few conceptual abstractions as possible. In the case of D5, one can see from the eight regions shown above that {R2} is the *concreta* and can only represent an objectual abstraction: the concept is a pure bundle (§10) of intersections $(A \cap B \cap C)$, which does not have any conceptual abstraction. All other regions, however, show abstractions of at least one diagrammatic object.

§23. Convex and Concave Concepts. The degree of abstraction of a concept can be measured by how many convex and concave boundaries it has. If D5 is broken down into individual regions, a total of four levels can be seen: (1) {R2} is a *concretum* since it has only concave boundaries; (2) {R5}, {R6} and {R7} have two concave and one convex boundary and are therefore *1st level-abstracta*; i.e. they are a conceptual abstraction of {R2}, but more concrete than regions of higher levels; (3) {R1}, {R3} and {R4} have one concave and two convex boundaries and are thus *2nd level-abstracta*; i.e. they are conceptual abstraction of a concept of a lower level. (4) {R8} has only convex boundaries and is therefore the *highest level-abstractum*.

§24. Conceptual Clusters. Through the exact interpretation of D4 and D5 we are now able to create and read more complex diagrams with Intersections and Exclusions. As an example, we take D4 (D2+D2*) and add two more concepts: first, `flower-bearing` (similar to Fig. 2) and, second, a new one such as `scratched`. One possible diagram with 5 conceptual spheres and 14 regions may be D6 including 2 *concreta* (1. flower-bearing ∩ green ∩ tree; 2. green ∩ table ∩ scratched), 6 *1st level-abstracta* (1. flower-bearing ∩ green; 2. flower-bearing ∩ tree; 3. tree ∩ green; 4. green ∩ scratched; 5. green ∩ table; 6. scratched ∩ table), and 5 *2nd level-abstracta* and 1 *highest level abstracta*. Due to the lack of space, we have only given the positive relations (intersections) in this description of the regions. Furthermore, we stop at this point with the prospect of what more complex Schopenhauer diagrams look like.

4 Summary and Outlook

In Section 2 we have presented Schopenhauer's main principles of his theory of language. In Section 3 a reconstruction, specification and further development esp. of Fig. 2 (D5) was carried out. In this context, we were able to explain many of the principles listed in Section 2 again with the help of Schopenhauer diagrams. However, this does not mean that research on Schopenhauer diagrams for conceptual analysis is by any means complete. Many of Schopenhauer's principles, topics and theses have not been addressed or sufficiently explained here, e.g. the principle of Circle-Inclusion from §12, Schopenhauer's theory of language development, the theory of translation, and many more.

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